Analytical Models for Protected plus Permitted Left-Turn Capacity at Signalized Intersection with Heavy Traffic

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This paper presents theoretical analysis of left-turn operations under heavy traffic. When traffic demand is heavy, residual queues from previous cycles may occur and contribute to blockage of the left-turn bay by adjacent through traffic. Such blockage reduces the left-turn capacity and is a random phenomenon. An improved model for protected left-turn capacity is first proposed, taking into account the influence of residual queues and blockage by through traffic. In addition, the classic model always overestimates the potential permitted left-turn capacity when opposing traffic is heavy. The potential permitted left-turn capacity is then modeled in a probabilistic way, and a simple estimation is suggested by investigating its relation to the classic model. Furthermore, because left-turn capacity may vary in response to a blockage, the coefficients of variance for protected left-turn capacity and for queue clearance time are introduced to evaluate the protected left-turn timing plan. The developed analytical models are validated with CORSIM simulations.

One of the most important issues of a signal timing plan is the left-turn treatment. In light traffic demand, the left-turn capacity can be easily determined by the effective green time, the saturation flow rate, and the opposing through traffic. However, in very heavy traffic demand, the situations become much more complicated. A common problem is left-turn blockage by the through traffic in the adjacent lane, particularly when the length of the left-turn bay is short.

Besides the length of the left-turn bay, the residual queue also affects the left-turn capacity. During a leading protected left-turn operation, when the traffic is heavy, the queue in the adjacent through lane might not be cleared at the end of one signal cycle and hence the probability of blockage to the left-turn bay is increased. The protected left-turn capacity would be reduced if the blockage to the left-turn bay occurs. Furthermore, the interaction between permitted left-turn vehicles and opposing through vehicles is complicated. All these factors lead to difficulties for obtaining left-turn capacity.

In the current *Highway Capacity Manual* (HCM) (*1*), the protected left-turn capacity is estimated by the product of the saturation flow rate of the left-turn movement and the percentage of protected green time during one cycle; the permitted left-turn capacity is calculated by the product of the potential left-turn capacity (filter saturation flow of permitted left turn) and the percentage of the unblocked green time during one cycle. The drawback of such a method is that it does not consider the probability of blockage to the left-turn bay and the residual queue, which results in an overestimated left-turn capacity.

Therefore, it is necessary to investigate the left-turn capacity at heavy-traffic intersections. The first part of this study proposes an improved model to estimate the leading protected left-turn capacity by considering the effect of residual queues and the probability of blockage to the left-turn bay. The second part of this study deals with permitted left-turn capacity. By analyzing a probabilistic model of potential left-turn capacity, it is shown that the methods used in the classic model and the HCM overestimate the potential left-turn capacity; an improved simple method is therefore recommended. In the end, the proposed left-turn capacity is validated by CORSIM simulations.

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Transportation Research Record: Journal of the Transportation Research Board, No. 2192, Transportation Research Board of the National Academies, Washington, D.C., 2010, pp. 177–184.
DOI: 10.3141/2192-17

LITERATURE REVIEW

In previous studies, the most-used theoretical capacity model for pretimed intersections follows the queueing model proposed by Beckmann et al. (*2*). It was assumed that arrivals or departures of vehicles may occur only during a set of constant time intervals of unit length. Such models work well without left-turn traffic. However, it is difficult for one to attempt to deal with left-turn traffic.

Later, Messer and Fambro studied left-turn capacity under different left-turn signal phases and presented a model for left-turn capacity that was reduced by left-turn blockage (*3*). Chang et al. explored the permitted left-turn saturation flow rate (*4*). They proposed a model containing multiplicative adjustment terms for the relation between the permitted flow rate and all other associated factors and estimated the parameters with log-linear regression. Tian and Wu studied the intersection capacity with a short right-turn lane (*5*). They proposed a probabilistic model for right-lane blockage and attempted to find the relation between the expected number of vehicles in the right lane and the proportion of through traffic.

Kikuchi et al. (*6*) analyzed left-turn lane lengths at signalized intersections by considering the probability of left-turn lane overflow and the blockage to the left-turn lane. A discrete probabilistic model was proposed to estimate the queue lengths of left-turn vehicles. From this model, a proper left-turn length could be recommended according to traffic and operation conditions, such as signal timing, left-turn demand, and through demand. Kikuchi et al. (*7*) also developed another method to analyze left-turn queue length of dual left-turn lanes under the assumption of Poisson arrival patterns for both through and left-turning traffic. However, they did not consider the residual queue for the left-turn or through movement.

Qi et al. (*8*) applied a discrete-time Markov chain model to analyze left-turn queue lengths at signalized intersections. They considered vehicles arriving during the red phase and the residual queue. Such a model was also applied to evaluate the length of left-turn lanes in the Houston, Texas, district (*9*). This is an innovative modeling approach because the effect of the residual queue and dynamics of fluctuations in through and left-turn traffic were incorporated into the model. However, the model-based Markov chain could not explicitly exploit the relation between the residual queue and the length of the left-turn queue.

Zhang and Tong proposed a probabilistic model for protected left-turn capacity at a signalized intersection with a short left-turn bay (*10*). With this model, they estimated the probability of the left-turn bay blockage and spillback. The model performed very well for a normal arrival rate. However, when the arrival rate turned heavy, the model might have had some problems because of the possibility of some residual queues in the adjacent through lane during the beginning of one signal cycle, and this factor was not considered in the model.

RESEARCH OBJECTIVES

This research studied a signalized intersection with a single left-turn bay under heavy traffic flows. First, the left-turn capacity was investigated during operation of the leading protected left turn. More specifically, the influence of residual queues and the effect of blockage by through traffic were investigated in this configuration. Accordingly, an model for improved protected left-turn capacity was developed on the basis of the model proposed by Zhang and Tong (*10*). Then, for the permitted phase, the potential left-turn capacity was modeled in a probabilistic way, and the relation to the classic model was presented. Furthermore, because the protected left-turn capacity may vary in response to the blockage and the queue clearance time may vary in a permitted left-turn phase, the variances of protected left-turn capacity and of queue clearance time were introduced to evaluate the protected left-turn timing plan and the length of the left-turn bay. The following steps show how the capacity model is derived:

- Calculate the through residual queue during one cycle, and then modify and calculate the probabilities of blockage of the left-turn bay by through traffic.
- Calculate the variance of protected capacity on the basis of the expected number of vehicles in the left-turn bay when a blockage takes place in the leading left-turn phase. The variance of protected left-turn capacity is introduced to describe the fluctuation of the number of vehicles in the left-turn bay for each cycle.
- Present a probabilistic model for potential permitted left-turn capacity, and find the relation to the classic model. A simple estimation method for potential permitted left-turn capacity is then suggested.
- The coefficients of variance for protected left-turn capacity and for opposing queue-clearance time in the permitted left-turn phase are introduced for evaluation of a left-turn timing plan with heavy traffic.

MODEL FOR LEFT-TURN CAPACITY

This section presents modeling elements related to left-turn capacity under heavy traffic. For protected left-turn operations, a residual queue at the end of the green period likely leads to more blockage, and this phenomenon is analyzed first. The blockage of the entrance to the left-turn lane due to insufficient length of the left-turn lane and its impact on the capacity is then modeled. A permitted left turn can add capacity to the left-turn movement. This part of the capacity is modeled theoretically with consideration of the interactions of left-turn vehicles with opposing through vehicles.

Improved Model for Protected Left-Turn Capacity Under Heavy Traffic

Residual Queue

To characterize protected left-turn capacity under heavy traffic, one needs to evaluate the probability of blockage due to through traffic and the number of vehicles in the left-turn bay when the blockage occurs. When the demand of through traffic is large enough, queue clearance time should be large as well. When green time is insufficient, residual queues will remain at the end of the previous signal cycle. On one hand, this does not mean that residual queues will occur at the end of each cycle. On the other hand, if they take place, they will lead to a higher probability of blockage to the left-turn bay in the next cycle. Therefore, it is important to determine the distribution or expected length of residual queues.

During the permitted left-turn phase, vehicles wishing to make left turns have to wait for a sufficient gap between two successive opposing through vehicles. Therefore, one can ignore the influence of left-turn vehicles to opposing through traffic. Hence, the residual-queue problem of through traffic can be converted to the case of a one-way intersection that has already been studied by previous authors [see, for example, Darroch (*11*), Newell (*12*), and Broek (*13*)]. In this section, only the isolated intersection is considered and some results for modeling residual queues are introduced.

To obtain easily computable expressions for through residual queues, it suffices to consider the queue as a random continuous flow. $Q_{TH}(t)$ denotes the number of vehicles in the through traffic queue at time t after the commencement of a red period. Then, the distribution of $Q_{TH}(0)$ is

$$F_Q(y) = \Pr\{Q_{TH}(0) \le y\} \quad (1)$$

where $F_Q(y)$ is the probability of the length of the queue being less than y at the start of the red time. Then,

$$F(x) = \Pr\{v_R R + v_g g - sg < x\} \quad (2)$$

where

v_R = through arrival rates during red time R,
v_g = through arrival rates during green time g, and
s = saturation flow rate.

The expression $v_R R + v_g g - sg$ can be read as the arrivals less the departures (if available) during a complete signal cycle. For a continuous model, the queue length at time $R + g$ [i.e., $Q_{TH}(R + g)$] is equivalent to $\max\{0, Q_{TH}(0) + v_R R + v_g g - sg\}$. Obviously, the distribution for $Q_{TH}(0)$ is the same as the distribution for $Q_{TH}(R+g)$. Thus, as suggested by Newell (*12*), comes the following relationship:

$$F_Q(y) = \int_0^\infty F_Q(x)\,dF(y-x) \tag{3}$$

Following the work of Newell (*12*), one can get a crude solution for Equation 3 and the expected length of residual queue:

$$E(Q_{TH}(0)) = \int_0^\infty x\,dF_Q(x) \approx \frac{Iq}{2s}\left[\frac{s(R+g)}{sg-q(R+g)}\right] \tag{4}$$

where q is the average arrival rate during the entire cycle and I is an appropriate variance-to-mean ratio for arrivals and departures (*12*). Broek et al. (*13*) also gave a review of traffic light queues and a new approximation to $E(Q_{TH}(0))$. Because Equation 4 is based on the continuous model, $E(Q_{TH}(0))$ cannot necessarily be an integer. Thus, $E(Q_{TH}(0))$ is rounded up to the closest integer of upper bound:

$$N_q = \left[E(Q_{TH}(0))\right] \tag{5}$$

It also assumed that the length of the residual queue is not enough to block the left-turn bay. Equation 4 is valid under very heavy traffic conditions. This next section reviews heavy traffic conditions (e.g., the arrival rate close to the departure rate). Otherwise, it is reasonable to ignore the occurrence of the residual queues when left-turn capacity is estimated.

Protected Left-Turn Capacity

It is assumed that the length of the left-turn bay is N normal vehicles. As a transitional area usually can contain two vehicles between the left-turn bay and the adjacent through lane, the blockage by through vehicles needs at least $N+2$ through vehicles arriving at the intersection in the adjacent through lane. To count the influence by residual queues, it is reasonable to imagine that the length of queue in the adjacent through lane varies from the residual queue length N_q to $N+2$ when a blockage occurs. One needs to be very careful when he or she uses the stationary solution of the length of residual queue. Under heavy traffic conditions, the traffic flow is in fact time-dependent; thus, the stationary equilibrium might not be attained. However, for the real application purpose, such equilibrium can help one get crude approximation results.

The probability of blockage takes place as long as there are $N+2-N_q$ through vehicles arriving in the adjacent lane and no spillback occurs. The associating probability can be estimated as

$$P_{\text{block}} = \Pr\left(\{X_T \geq N+2-N_q\} \cap \{X_{LT} \leq N+2\}\right) \tag{6}$$

where X_T and X_{LT} represent the number of vehicles in the adjacent through lane and in the bay, respectively, during the red time before the commencement of leading protected left-turn signal. This blockage probability is easily calculated once the arrival distribution is known. Both the through and left-turn arrival distributions should be carefully chosen, because the arrival pattern would be different at low and heavy traffic. For a real-world application, an empirical distribution estimated from the historical data can be used instead of mathematical distributions.

Based on the above results, the expected number of vehicles in the left-turn bay when blockage occurs can be modified (*10*):

$$E(X_{LT}) = \sum_{x=0}^{N+2} x f(x) \tag{7}$$

where $f(x)$ is the probability that x left-turn vehicles arrive in the bay before blockage. It follows a negative binomial distribution with this formula:

$$f(x) = \binom{x+N+1-N_q}{N+1-N_q}(1-p_1)^x\, p_1^{N+2-N_q} \tag{8}$$

where p_t denotes the proportion of the through traffic. Although use of a negative binomial distribution may lose some accuracy, the result can be seen as a good approximation for a real-world application.

Therefore, the protected left-turn capacity can be calculated by combining the expected number of vehicles during blockage and the number of left-turn vehicles in the unblocked normal condition:

$$c_{\text{protected}} = nP_{\text{block}}E(X_{LT}) + (1-P_{\text{block}})\frac{s_{LT}g_{LT}}{C} \tag{9}$$

where

n = number of cycles in peak hour,
s_{LT} = saturation flow rate for protected left-turn movement,
g_{LT} = effective green interval, and
C = cycle length.

This probabilistic equation indicates that the protected left-turn capacity is a random variable. Hence, its features are characterized here. This issue will be discussed in a later section.

A residual queue problems also exists if the arriving left-turn rate is so heavy that some vehicles cannot leave the left-turn bay at the end of the protected green time. However, this problem cannot be solved by following procedures similar to those in previous studies, and few studies exist about it. Therefore, this problem is left to future work.

Model for Permitted Left-Turn Capacity

Usually two steps are needed to compute the permitted left-turn capacity. The first one is to estimate the potential permitted left-turn capacity (sometime referred to as "filtered saturation flow rate"); the second is to estimate the blocked time.

The expected blocked time is easy to determine, however, the difficulties arise when one tries to estimate the potential left-turn capacity. To do this one has to consider the interaction between the left-turn traffic flow and the opposing through traffic. Unless the passing headway T is larger than the critical gap T_C, no left-turn vehicle can enter the through-traffic gap and make a left turn. The well-known procedure used in the HCM of estimating potential capacity for left-turn originates from Drew's deduction (*14*). The basic idea is consideration of the probability of a large gap in which a vehicle can enter or merge into the through traffic within 1 h. Drew's deduction computes the probability of every passing headway between any $T_C + iH_f$ and $T_C + (i+1)H_f$, where H_f represents follow-up headway and i is an integer. However, this calculation is valid only when all arriving through headway are independent from each other. If the headway of every arriving vehicle can vary without dependence, the total time of all arriving vehicles within 1 h could be infinite. Hence, as will be shown, the capacity calculated by the HCM might be overestimated. This section will first focus on the potential capacity that contributes the most part of inaccuracy for the eventual left-turn capacity.

To estimate an accurate capacity, the dependence of consecutive headways within a limited time should be taken for consideration. Because two arriving vehicles are neither too close to each other nor too far from each other during the peak hour, the headway should have minimum and maximum values denoted by $H_{\min}$ and $H_{\max}$, respectively. For the following analysis, v_g and v_R denote the arrival flow rate [in vehicles per hour (vph)] during the green and red times, respectively. Then $C(N, T)$ represents the potential number of vehicles that could make left turns when there are N opposing through vehicles passing the intersection within a period T (in seconds). In this definition, $C(v_g, 3{,}600)$ is the potential left-turn capacity.

One can suppose that, on average, N opposing through vehicles are arriving during T. One can further consider that the headway H_1 of the first two consecutive opposing through vehicles passing intersection. H_1 can vary from $H_{\min}$ to $H_{\max}$. Thus, if the probability that H_1 is less than x, is denoted by $F_H(x) = \{H_1 < x\}$, then $F_H(x) = 0$ when $x < H_{\min}$ and $F_H(x) = 1$ when $x > H_{\max}$. Hence, by the total probability theorem, it is easy to get the following equation:

$$C(N,T) = \int_0^{T_C} C(N-1, T-x)\,dF_H(x) + \sum_{i=0}^{\infty} \int_{T_C+iH_f}^{T_C+(i+1)H_f} \big(C(N-1, T-x) + (i+1)\big)\,dF_H(x) \qquad (10)$$

One can compute $C(N, T)$ by recursive Equation 10 with some appropriate boundary conditions. However, the computation will be expensive. Here, this equation is used only to address some problems when the permitted left-turn capacity is calculated. If it is assumed that $C(N-1, T-x) \approx C(N, T)$ the solution of $C(N, T)$ is as shown in Equation 11:

$$\big(1 - F_H(H_{\max})\big) C(N,T) = \sum_{i=0}^{\infty} (i+1) \Pr\{T_C + iH_f < H_1 < T_C + (i+1)H_f\} \qquad (11)$$

If the expression $[1 - F_H(H_{\max})]^{-1}$ is equal to v_g, Equation 11 becomes Drew's equation. Thus, when one assumes Poisson arrival, the expression is also identical to the equation for the potential left-turn capacity in Appendix C of Chapter 16 in the HCM (*1*). From the above analysis, the conclusion can be made that the HCM method overestimates the potential left-turn capacity. Because it is know that the headway is less than $H_{\max}$ for most cases, a simple way to modify potential capacity reads

$$c_{\text{potential}} = v_g \sum_{i=0}^{1<H_{\max}} (i+1) \Pr\{T_C + iH_f < t < T_C + (i+1)H_f\} \qquad (12)$$

However, under heavy opposing traffic conditions, determining the probability $H_{\max}$ is a difficult task, and, according to numerical simulation in the next section, the better way may be to use three or four summarizing terms in Equation 12 (i.e., summarizing terms from $i = 0$ to $i = 3$ or 4). Although this procedure may not be exactly accurate and one still needs to use real-world data to calibrate the order of the model, the procedure presents a way to find an approximation or a bound to the real potential left-turn capacity when opposing traffic is not light. Because of the randomness of arrivals and uncertainties of the behaviors of drivers, it is hard to get the exact value of the capacity. Hence, such approximation or bound of potential left-turn capacity would be important for a real-world application. An example will be shown later, and this issue will be left to future work. When opposing through traffic is light, the HCM procedures can be seen as a good approximation (see a later section). In this case, the permitted left-turn capacity can be estimated with a modified HCM method (*1*):

$$\begin{aligned} c_{\text{potential}} &= v_g \frac{e^{-v_g T_C/3600}}{1 - e^{-v_g H_f/3600}} \\ c_{\text{permitted}} &= \frac{g_u}{C} \times c_{\text{potential}} \\ g_u &= g - g_q \\ g_q &= \frac{v_R \times R}{s - v_g} \end{aligned} \qquad (13)$$

where

g_u = unblocked green time,
g_q = blocked time, and
R = red interval.

Evaluation of Timing Plan

How to evaluate a plan for leading left-turn timing is an important problem for real-world applications. On one hand, the time setting for a protected left-turn phase should avoid blockage of the left-turn bay by the adjacent through lane; on the other hand, the time setting for the permitted left-turn phase should provide enough time for the through-queue clearance.

First, due to the uncertainty of blockage, the maximum volume of vehicles that can make left turns must vary from time to time. Accordingly, it is reasonable to introduce the variance of capacity for the protected left-turn phase to characterize such fluctuations. The variance of capacity can be estimated as

$$V_{\text{potential}} = n^2 P_{\text{block}}^2 \operatorname{Var}(X_{\text{LT}}) \qquad (14)$$

where $\operatorname{Var}(X_{\text{LT}})$ is the variance of the number of vehicles in the left-turn bay when the blockage occurs; it is easy to compute by using Equation 7. If this variance is relatively large, it indicates that the timing plan for the protected left turn is bad or the length of the left-turn bay is too short under present traffic conditions. Therefore, the coefficient of variance $\sqrt{\operatorname{Var}(X_{\text{LT}})}/c_{\text{protected}}$ can serve as an indicator for evaluation of the plan for protected left-turn timing.

Second, to evaluate the timing plan for a permitted left-turn signal, one needs to estimate the variance of the clearance time for opposing through queues (blocked time). If the variance is too large, it implies that there are opposing through queues at the end of the permitted red left-turn phase and the vehicles cannot make left turns. To describe the blocked time, classic models and the HCM procedure both depend on the analogue to a deterministic continuous fluid model. However, it is not enough because the blocked time varies cycle by cycle. A better way is to borrow the results from random continuous queueing models. By using the notations in previous sections, the variance of clearance time for through queues can be calculated by standard queueing theory (*15*):

$$V_{\text{clearance}} = \frac{s v_g}{\left(s - v_g\right)^3} \times R \tag{15}$$

In Equation 15, the values of v_g and s should be converted equivalently into the values during one cycle. Similarly, evaluation of the plan for protected left-turn timing can use the coefficient of variance $\sqrt{V_{\text{clearance}}}/E_{\text{clearance}}$, in which the expected clearance time $E_{\text{clearance}}$ can be calculated in a normal way.

RESULTS AND VALIDATION

This section first examines the issue of potential permitted left-turn capacity. It then uses simulation results to validate the combined left-turn capacity model.

Potential Permitted Left Turn

To get the potential permitted left-turn capacity, a traffic simulation model, which consists of one unsignalized intersection, was set up in CORSIM. Only left-turn vehicles were assigned to one direction and only through vehicles with six different flow rates (from 1,000 to 1,500 vph) were assigned to the opposing direction. For each scenario of opposing flow rate, eight simulation runs were done as a result of the stochastic nature of CORSIM. The potential left-turn capacity can be obtained by increasing the left-turn traffic demand until the throughput reaches its maximum. To compare the model with the HCM equation, the critical gap and follow-up headway were set to 5.0 and 2.5 s, respectively. The reason for these setting is that TRAF-NETSIM uses 5.0 s as the default value for the critical left-turn gap (*4*). From the results shown in Figure 1, it is obvious that the HCM results overestimate the potential left-turn capacity. But when the opposing through traffic gets relative light, say, less than 1,100 vph in this case, the gap becomes smaller than the gap when opposing through traffic is heavy. In the real world, because of the randomness of arrivals and the uncertainties of the behaviors of drivers, it is extremely hard to obtain the exact value of the potential left-turn capacity. As stated earlier, Equation 12 can be used to obtain an approximated potential left-turn capacity. If the summarization terms in Equation 12 are set to four for an opposing through traffic rate of 1,000 vph, three for an 1,100-vph traffic rate, and two for other traffic rates, the estimated results well match the CORSIM results. However, such a procedure is very simple, and more precise models may still be needed.

Model Validation

Simulation Data

A two-lane isolated signalized intersection was set up in CORSIM and generated simulation data for evaluation of the developed probabilistic model. The intersection operates with the strategy of a leading left turn and has two 12-ft lanes with one left-turn bay, all passenger cars, no parking, and no pedestrians. The length of the left-turn bay was selected as a variable in the capacity calculation. The main input data for the capacity calculation are as follows:

- Number of through lanes, 2;
- Protected left-turn green, 13 s;
- Through vehicle green, 50 s;
- Through vehicle red, 63 s;
- Total cycle length, 117 s;
- Change interval for each phase, 4 s;
- Through vehicle volume, 1,550 veh/h;
- Left-turn volume, 388 veh/h;
- Mean discharge headway, 1.9 s;
- Opposing through vehicle volume, 850 veh/h; and
- Opposing through vehicle arrival type, 3.

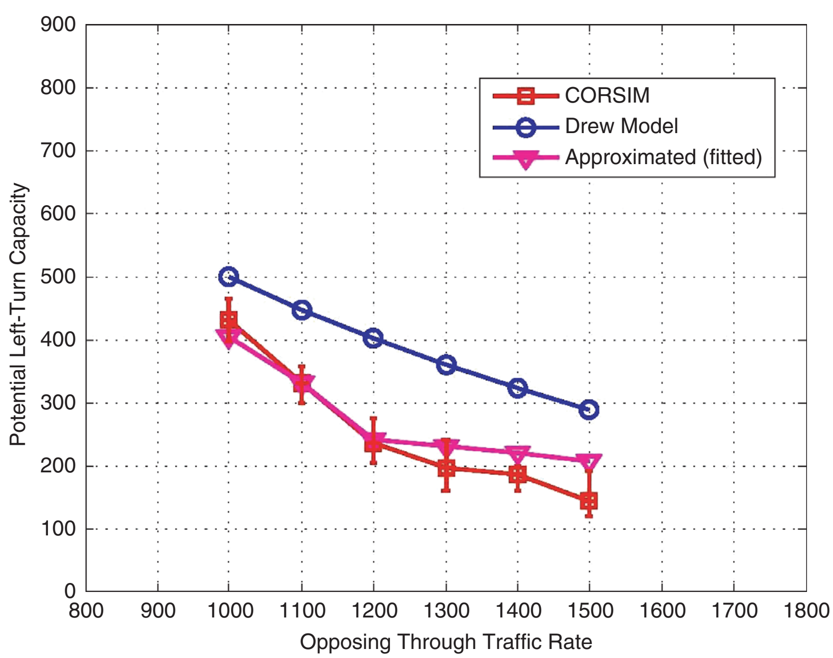

FIGURE 1 Examination of model for potential left-turn capacity.

For each of the eight length scenarios (i.e., the left-turn bay length from five to 12 vehicles, five simulation runs of 15 min) were done by changing the random-number seeds in CORSIM. During each run, the authors managed to obtain the left-turn results from CORSIM by increasing the total demand until the output reached its maximum.

Validation of Left-Turn Capacity Model

The probabilistic model was used for protected left-turn capacity, and the modified HCM was used for permitted left-turn capacity because of the light opposing through traffic. In relation to protected left-turn capacity, the first thing to do is to estimate the length of the residual queue. It is appropriate to set up 51 s for green time in Equation 4 because drivers may use the first second of yellow time to pass the intersection. Obviously, the variance-to-mean ratio I for arrivals and departure is bounded by a certain value because of the rate of underdispersed distributed traffic flow. This value can also be estimated from the real-world or simulation data. It was supposed that vehicles arrive at the intersection with equal distribution for each through lane. Accordingly, the expected residual queue length was estimated to be one vehicle for each lane and was bounded by three vehicles. The calculation of residual queue length would be sensitive to the arrival rate and the effective green time. In the real world, the residual queue lengths may vary greatly cycle by cycle, especially when traffic is heavy. Thus, knowing the bound of residual queue length is also important for an application. From this residual queue estimation, it is easy to calculate protected left-turn capacity after obtaining the average number of left-turn vehicles blocked in the left-turn bay by the adjacent through traffic and the probability of blockage. Figure 2 shows the relationship between the length of the left-turn bay and the probability of blockage with residual queue lengths of one, two, and three vehicles in the adjacent through lane. To evaluate the protected left-turn timing plan, the coefficient of variance for protected capacity is calculated for each left-bay length scenario when the expected length of the residual queue is one vehicle. Figure 3 shows the results. This figure shows that the coefficient of variance is relatively large for a short left-turn bay. This indicates that the number of discharging left-turn vehicles fluctuates intensely during the protected left-turn phase. Such a phenomenon implies that the timing plan for the short left-turn bay cannot work well for the given traffic conditions.

In relation to the permitted left-turn capacity, the modified HCM method was used because of the light opposing through vehicles. The critical gap and the follow-up headway used are 4.5 and 2.5 s, respectively, which have also been used in the current version of HCM. Finally, combining the results of the protected and permitted left-turn capacity calculation, the total left-turn capacity can be estimated to be from 285 vph when the length of the left-turn bay is five vehicles to 376 vph when the length of left-turn bay is 12 vehicles. Figure 4 illustrates the left-turn capacity compared with the simulated results with respect to the length of the left-turn bay. To illustrate the range of simulation results, the minimum and maximum values for each scenario are plotted as error bars in Figure 4 associated with the mean value of the simulated results. In the figure, the left-turn capacity results from the HCM methods are also shown and remain a constant 378 vph for different scenarios. Obviously, the HCM methods overestimate the left-turn capacity, especially when the length of the left-turn bay is not sufficiently long. Because of the uncertainty of left-turn spillback, the residual queues may not be equally distributed in each through lane, and this phenomenon occurs frequently at short length left-turn bays (i.e., the bays with five to six vehicles in length). The interaction between left-turn and through vehicles is thus complicated at the left-turn bays with very short lengths, and the proposed model may underestimate left-turn capacity. The proposed model overestimates left-turn capacity when the left-turn bay is long. The reason this happens is that the residual queue would occur when the left-turn bay gets longer. This issue will receive attention in future work. When the stochastic nature of simulation is considered, the results show that the proposed probabilistic

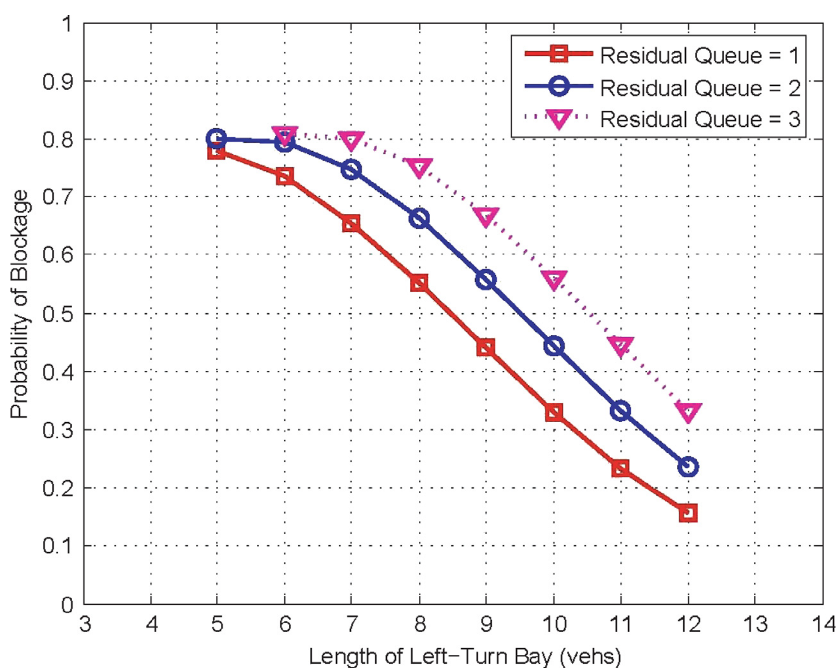

FIGURE 2 Influence of residual queue length on probability of blockage.

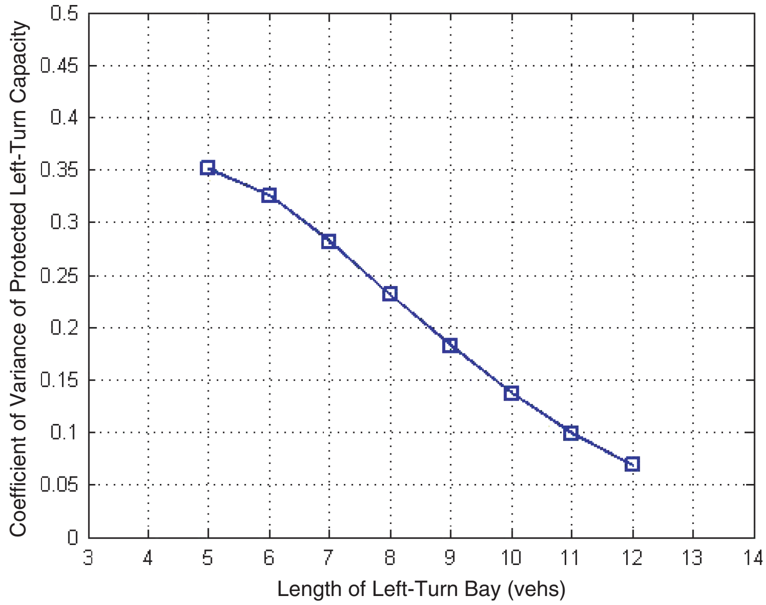

FIGURE 3 Coefficient of variance for protected left-turn capacity when residual queue is one.

model well reflects the left-turn capacity due to the residual queue under heavy traffic.

CONCLUSION AND FUTURE WORK

This paper studied the left-turn capacity with heavy traffic under a leading protected left-turn signal. This problem is inherently related to the problem of residual queues at a signalized intersection. The complicated features include the expected length of residual queues, the probability of blockage to the left-turn bay, and the interaction between left-turn traffic and opposing through traffic. Considering these factors, the authors proposed a probabilistic left-turn capacity model and investigated the potential permitted left-turn capacity model under heavy traffic. The model for protected left-turn capacity describes the impact of adjacent through residual queues on blockage of the left-turn bay, and the analysis of permitted left-turn capacity indicates the inaccuracy of the HCM model when traffic is heavy. Although the simulation results support the proposed analysis and models, the problems with left-turn capacity still need to be investigated further. The proposed model does not consider the residual queues of the left-turn traffic, and this issue will be studied in the future. Future work will further study the interaction between the left-turn and adjacent through traffic with heavy traffic under different

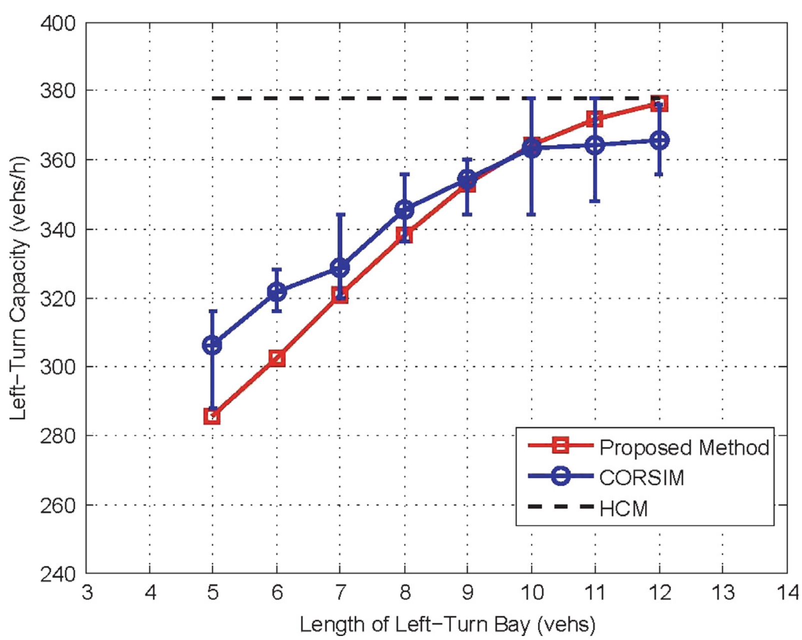

FIGURE 4 Validation of probabilistic model for left-turn capacity.

operating strategies. The model will be validated and applied to field data. And issues such as left-turn capacity for general arrival patterns and permitted left-turn capacity will also be further investigated.

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The Traffic Signal Systems Committee peer-reviewed this paper.