Modeling Delay During Heavy Traffic for Signalized Intersections with Short Left-Turn Bays

Kai Yin, Yunlong Zhang, and Bruce X. Wang

This paper develops theoretical delay models for protected left-turn operations at a pretimed signalized intersection during heavy traffic. When the through traffic demand is heavy, residual queues from the previous cycle may occur and elevate the probability of blockage to the left-turn bay, leading to increased delay for left turns. A probabilistic left-turn delay model based on the queuing diagram is proposed for a leading left-turn operation; the influence of residual queues and blockage by the through traffic are taken into account. When the left-turn demand becomes heavy, the left turns may spill back to block the through traffic, resulting in through traffic delays. Through traffic delay is modeled probabilistically on the basis of the analysis of left-turn bay spillback for a lagging protected left-turn operation. The left-turn delay models are validated through carefully designed simulation studies using VISSIM and the results are compared with those from the *Highway Capacity Manual* (HCM) delay model, which does not consider blockage or spillback situations. The proposed delay models can be used to replace the uniform delay term in the HCM model for high-demand situations when the left-turn operation is affected by spillback and blockage.

Almost five decades have passed since Webster published his remarkable work on traffic signal setting and traffic delay (*1*). Signal timing remains critical to traffic engineers, especially in populous cities. One of the most important issues of signal timing is the left-turn treatment. Usually a protected left-turn signal phase, before (leading) or after (lagging) the through signal, is applied to a signalized intersection with high traffic demand. The traffic delay is used to evaluate the signal operations. In light traffic demand, the average delay can be easily determined by treating left-turn and through traffic independently. However, when the demand is heavy (e.g., close to capacity), the situation becomes much more complicated because residual queues are a problem and there is interaction between left-turning and through vehicles.

A common problem for leading left-turn operation is the blockage to left-turning vehicles by through traffic, especially at intersections with short left-turn bays. During peak hours, some vehicles in the through lane might not be able to depart at the end of one cycle, resulting in an increased probability of left-turn blockage. In turn, the blocked left-turning vehicles may also delay the ability of the through traffic to enter the intersection. Those problems may not exist during a lagging left-turn operation because left-turning vehicles tend to spill out of the bay under heavy traffic. In this case, the through capacity is reduced, leading to increased total delay. All of these factors contribute to the difficulties in estimating average delay. Furthermore, these issues are not included in the considerations of the methods in the *Highway Capacity Manual* (HCM) (*2*).

Therefore, it is necessary to investigate the traffic delay during heavy traffic at pretimed intersections with short left-turn bays. The objective of this paper is to develop probabilistic models to correct the uniform delay model in the HCM. The first part of this study proposes a probabilistic model to estimate the average delay per vehicle with leading protected left-turn operations by considering the probability of blockage to the left-turn bay and a residual queue of through traffic. The errors that might arise in the method are discussed. The second part of this study deals with traffic delay under lagging left-turn operations. By considering the probability of left-turn spillback, researchers are able to introduce an improved delay model for through traffic to replace the uniform delay model in the HCM. A calibrated VISSIM simulation is used to compare the proposed model with the HCM methods.

METHOD OF *HIGHWAY CAPACITY MANUAL*

In Chapter 16, the HCM provides procedures for estimating delay of lane groups at signalized intersections (*2*). The average delay per vehicle for a given lane group is computed as a sum of three parts: uniform control delay, assuming uniform arrivals; incremental delay to account for random arrival effect; and initial queue delay. For left-turn protected operations, the HCM methods treat left turns and through traffic independently. It does not adjust for the left-turn blockage. Nor does the method adjust the influence of left-turn spillback on through traffic. If the arrival volume-to-capacity ratio is less than 1.0, the delay calculations are not different in various left-turn protected treatments (but are different in protected and permitted operations). However, as discussed in the following sections, during heavy traffic the interactions between left-turning and through vehicles result in different delays between leading and lagging left-turn operations, affecting the uniform delay model in the HCM methods.

REVIEW OF LITERATURE

There is extensive literature on estimating delay for through traffic at signalized intersections. Perhaps one of the most important theoretical delay models for pretimed intersections was proposed by

Department of Civil Engineering, Dwight Look College of Engineering, Texas A&M University, 3136 TAMU, College Station, TX 77843-3136. Corresponding author: K. Yin, k-yin@ttimail.tamu.edu.

Transportation Research Record: Journal of the Transportation Research Board, No. 2257, Transportation Research Board of the National Academies, Washington, D.C., 2011, pp. 103–110.
DOI: 10.3141/2257-12

Beckmann et al. (*3*). It is assumed that arrivals or departures may occur only on a set of constant time intervals of unit length. Notable studies, including those by Newell (*4*), Darroch (*5*), and Leeuwaarden (*6*), used an analytical approach to study the queues at signalized intersections and traffic delay. Their work generally depended on some assumed discrete structure of stochastic process. In another pioneering work, Newell (*7*) used an approximation to obtain the average delay in a fairly general manner that could be applied to a broad range of processes. In general, these models work well without left-turn traffic.

However, there is little literature on theoretical models of average delay for an intersection with left-turn traffic. Wang and Benekohal (*8*) considered the effects of platoon arrival rate and platoon duration under protected and permitted left-turn operations at signalized intersections. After distinguishing two kinds of arrival rates, one for platoon and one for nonplatoon, they developed the uniform delay model based on an arrival departure diagram. Some studies on left-turn traffic are about the capacity and left-turn lane design, with emphasis on blockage, spillback, and left-turn residual queues. Although the researchers do not intend to evaluate delay, their results help to understand the interactions between left-turn and through traffic.

Messer and Fambro studied the left-turn capacity under different left-turn signal phases and presented a model for left-turn capacity that was reduced by left-turn blockage (*9*). Tian and Wu studied the capacity of intersections with short right-turn lanes (*10*). They proposed a probabilistic model for right-lane blockage and attempted to find the relationship between the expected number of vehicles in the right lane and the proportion of through traffic.

Kikuchi et al. (*11*) analyzed left-turn lane lengths at signalized intersections by considering the probability of left-turn lane overflow and blockage of the left-turn lane. A discrete probabilistic model was proposed to estimate the queue lengths of left-turn vehicles. Kikuchi et al. (*12*) also developed a method to analyze the left-turn queue length of dual left-turn lanes under the assumption of Poisson arrival patterns for both through and left-turn traffic. However, they did not consider the residual queue for the left-turn or through movement. Qi et al. (*13*) considered the overflow of left-turn traffic and applied a discrete-time Markov chain model to analyze left-turn queue lengths at signalized intersections. Such a model was also applied to evaluate the length of left-turn lanes in Houston, Texas (*14*).

Zhang and Tong proposed a probabilistic model for protected left-turn capacity at a signalized intersections with short left-turn bays (*15*). They estimated the probability of the left-turn bay blockage and spillback. The model performs well for normal arrival rates. However, when the arrival rate becomes heavy, the model might have some problems, because there would be residual queues in the adjacent through lane during the beginning of a signal cycle. These residual queues are not considered in the model.

RESEARCH OBJECTIVES AND METHODOLOGY

This research studies the delay per vehicle under heavy traffic at a pretimed signalized intersection with a single left-turn bay. The focus is on the correction of the uniform delay term in the HCM methods. The framework is attributed to the previous work by Zhang and Tong (*15*). The authors assume the opposing through volume is high so that a permitted left-turn operation cannot be applied. First, the delay was investigated during a leading protected left-turn operation. More specifically, the probability of blockage by through traffic was calculated by estimating the through residual queue. Accordingly, a delay model with a leading protected left-turn operation was developed on the basis of the queuing diagram. Such a delay model for left-turn traffic could be used to replace the uniform delay term in the HCM methods when the adjacent through flow is heavy. The potential errors in the proposed method were also discussed. Then, for a lagging protected operation, the through traffic delay was modeled in a probabilistic way on the basis of the analysis of left-turn bay spillback. By combining the proposed models with the incremental delay and, if necessary, the initial queue delay in the HCM methods, one can obtain the control delay for an intersection under heavy traffic. An outline of the steps for deriving the delay models:

- Calculate the discrete probability of through residual queues by conversion from the continuous case, and then modify and calculate the probability of left-turn bay blockage under a leading protected left-turn signal.
- Calculate the probabilities of the number of vehicles in the left-turn bay during the blockage. These values help determine the left-turn delay.
- Propose a probabilistic model for left-turn delay on the basis of the analysis of the queuing diagram, and then use the proposed left-turn delay to correct the uniform delay term in the HCM methods.
- Calculate the probability of left-turn spillback from a bay by incorporating the influence of residual queues under a lagging protected left-turn signal.
- Propose the through delay model on the basis of the probability of left-turn spillback and then estimate the control delay by combining the incremental delay and initial queue delay.

DELAY MODEL FOR SIGNALIZED INTERSECTIONS

Delay Model for Leading Protected Left Turns in Heavy Traffic

During peak hours when the through traffic demand is high, the leading protected left-turn signal operation would lead to a situation that quickly queued adjacent through vehicles and blocked the left-turn bay. Such a phenomenon would occur especially for cycles during which the fluctuations in the number of arrivals are large. When blockage to the left-turn bay occurs, only the vehicles in the left-turn bay can depart during the protected left-turn phase, and the blocked left-turning vehicles are delayed until the next cycle. Obviously, those blocked vehicles increase the total delay at a signalized intersection.

Some critical points for the cases in this paper are clarified here. When blockage occurs during a red time, the consecutive green time for through traffic is generally long enough to allow blocked left-turning vehicles to enter the left-turn bay to wait for the next cycle. Although the studied situation is heavy traffic—not oversaturated but close to capacity—those blocked vehicles do not necessarily wait two or more cycles at the intersection. Meanwhile, the authors allow under heavy traffic the stochastic fluctuations in the number of arrivals to cause some through vehicles not to be served during some cycles, resulting in residual queues at the beginning of red signals. The residual queues would lead to a high probability of blockage in the next cycle. Conversely, if there are blocked left-turning vehicles, they would delay the following through vehicles from entering the intersection and, hence, contribute to the probability of through residual queue length, especially when the number of through lanes is limited to one or two. One may desire to estimate the delay by analytically establishing the relationship between residual queues and blockage of the left-turn bay. However, this way may lead to some mathematical difficulties and tedious algebra operations, leaving little meaningful guidance for practice. In this section, the authors adopt an easier treatment. The authors first directly estimate the residual queue length on the adjacent through lane on the basis of arrivals and then obtain the left-turn bay blockage probability. From these results, the total delay at the intersection can be obtained. Finally, the authors verify that the errors in such treatment should be small compared with total delay per cycle.

Leading Protected Left-Turn Delay

As in the study by Zhang and Tong (*15*), the authors assume the length of the left-turn bay to be N normal vehicles, and $N + 2$ adjacent through arrivals can block the left-turn bay because of the transitional area between the left-turn bay and the adjacent through lane. Denote the length of the residual queue on the adjacent through lane by the random variable N_q, and denote its probability by $\Pr(N_q = n)$. Now assume that $\Pr(N_q = n)$ is already known. (Its estimation will be discussed in the next subsection.) Because a blockage takes place as long as there are $N + 2 - N_q$ through vehicles arriving in the adjacent lane and no left-turn spillover occurs, the blockage probability can be estimated as

$$P_{\text{block}} = \sum_{n=0}^{N+2} \Pr\left(\left\{X_{\text{TH}} \geq N + 2 - n\right\} \cap \left\{X_{\text{LT}} \leq N + 2\right\}\right) \Pr\left(N_q = n\right) \quad (1)$$

where X_{TH} and X_{LT} represent the number of vehicles in the adjacent through lane and in the bay, respectively, during the red time before the start of leading protected left-turn signal. In general, the calculation of probability $\Pr(\{X_{\text{TH}} \geq N + 2 - N_q\} \cap \{X_{\text{LT}} \leq N + 2\})$ is not trivial.

Although the Poisson arrival assumption can help easily calculate this probability and has been used in several left-turn traffic studies (*12, 13*), it is known that heavy traffic does not follow the Poisson distribution. Perhaps one reasonable method for general cases is to use negative binomial distribution by considering the arrivals on the adjacent through lane as failures and those in the left-turn bay as successes during the through red interval. The blockage probability can be obtained accordingly by cumulating each discrete probability for the possible number of vehicles in the left-turn bay. The errors that may arise in this method are due to the fluctuations in the arrival number and the distribution of vehicles on the multiple through lanes. If an isolated intersection is considered, the Poisson distribution can be applied as an approximation.

When blockage occurs, no new vehicles can join the queue in the left-turn bay. However, the blocked vehicles still wait at the intersection during the red interval. It is more convenient to imagine a hypothetical queue that accumulates according to the left-turning arrivals after blockage, as indicated by the dashed curve around the shaded area in Figure 1. In doing so, one avoids estimating the beginning time of blockage, which leads to extra tedious calculation. Hence, before the start of the protected green time, one can use the normal method to calculate the uniform delay according to the shaded area under the queue curve shown in Figure 1. To calculate the delay for blocked vehicles during the left-turn green time, which is also shaded in Figure 1, one should first calculate the probability of the number of vehicles in the bay during blockage. The starting time of left-turning arrivals should also be considered, because the new left-turning vehicles can enter the bay only after the departure of the $N + 2$ adjacent through arrivals. Without additional knowledge, such time could be treated as zero, but one should keep this information in mind.

From the above analysis, the authors first estimate the probability $p(x)$ that x left-turning vehicles arrive in the bay before blockage. It follows a negative binomial distribution with the formula

$$p(x) = C_1 \sum_{n} \binom{x + N + 1 - n}{N + 1 - n} \left(1 - p_t\right)^x p_t^{N+2-n} \Pr\left(N_q = n\right) \quad (2)$$

where p_t denotes the proportion of through traffic and C_1 is a constant such that $\sum_{x=0}^{N+2} p(x) = 1$. If n in the summarization becomes larger than the length of the left-turn bay, the multiplier of the residual queue probability can be set to one. Because of the blockage, there will not be any residual queue in the left-turn bay. The total uniform delay for all left-turning vehicles per cycle, denoted by d_{LT}^T, can be calculated according to the following equations:

$$d_{\text{LT}}^T = P_{\text{block}} d_{\text{block}}^T + \left(1 - P_{\text{block}}\right) d_{\text{unblock}}^T \quad (3)$$

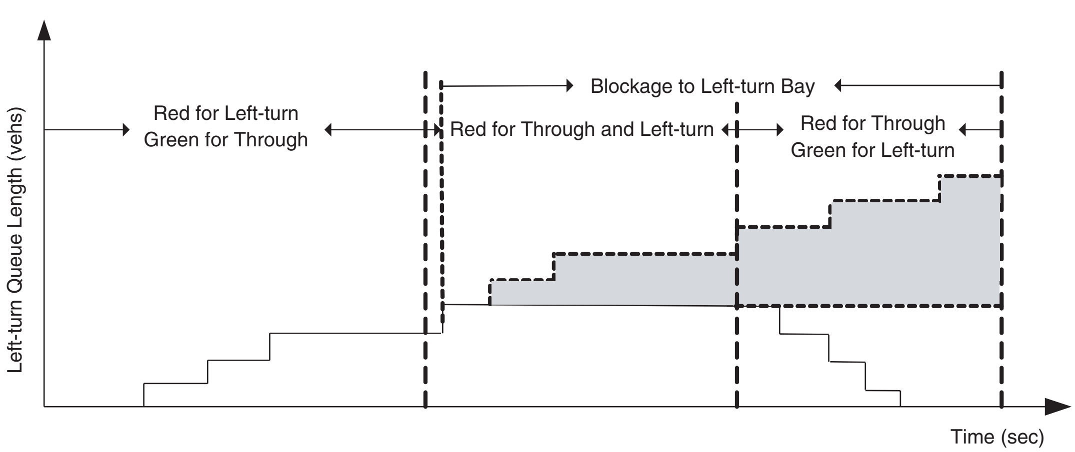

FIGURE 1 Realization of queue in left-turn bay when blockage occurs.

$$d^T_{\text{unblock}} = \frac{1}{2} r^2_{\text{LT}} v_{\text{LT}} + r_{\text{LT}} v_{\text{LT}} g_{\text{LT}} + \sum_{x=0}^{N+2} p(x)\left(\frac{1}{2}\frac{x^2}{s_{\text{LT}}} - g_{\text{LT}} x\right) + \frac{1}{2} g^2_{\text{LT}} v_{\text{LT}} \tag{4}$$

$$d^T_{\text{unblock}} = \frac{1}{2} \frac{v_{\text{LT}} r^2_{\text{LT}}}{\left(1 - v_{\text{LT}}/s_{\text{LT}}\right)} \tag{5}$$

where

- s_{LT} = saturation flow rate for protected left-turn movement (veh/s),
- x = number of left-turning vehicles arriving in the bay before blockage,
- r_{LT} = red signal time for left-turning vehicles,
- g_{LT} = protected left-turn green time, and
- v_{LT} = average left-turning arrival rate (veh/s).

Equation 4 is derived from estimating the shaded area (the imaged queue) and the area between the queue curve and time axis. Equation 5 is essentially the same, with the uniform delay in the HCM methods when the traffic condition is not oversaturated. It is apparent that the proposed average uniform delay model, Equation 3, is essentially a weighted combination of the blocked left-turning vehicles and others. To obtain the average uniform delay d_{LT} per vehicle, one can divide d^T_{LT} by the average number of arrivals per cycle.

$$d_{\text{LT}} = \frac{d^T_{\text{LT}}}{v_{\text{LT}} C} \tag{6}$$

where C is one signal cycle length. Equation 6 can be used to replace the uniform delay term d_1 in the HCM methods (*2*) to estimate the control delay for left-turn traffic.

Through Traffic Delay

The delay of through traffic contributes the most to total traffic delay at a signalized intersection. When the through traffic flow is close to saturation, some through vehicles that cannot depart in one cycle will be carried over to the next cycle as a residual queue. The importance of the residual queue to delay problems can be found in the literature [see, for example, work by Newell (*4*), Darroch (*5*), and Broek et al. (*16*)]. The estimation of residual queue length is relatively difficult and usually depends on the stochastic assumption of arrival structure. Heavy traffic is different from normal traffic because the residual queue may not be sensitive to the stochastic structure. Perhaps it suffices to use diffusion approximation to calculate the residual queue, and its errors should be of the order of the reciprocal for the number of arrivals during one cycle. To apply diffusion techniques, one can consult work by Newell (*7*). His results are used to estimate the through delay in this section.

For ease of presentation, the authors consider the multiple through lanes as one lane and denote by Q_{TH} the length of through queue right after the commencement of a red period. Let the distribution of Q_{TH} be

$$F_Q(y) = \Pr\{Q_{\text{TH}} \le y\} \tag{7}$$

Newell (*7*) derived $F_Q(y)$ and $E(Q_{\text{TH}})$ as the following equations:

$$F_Q(y) = 1 - e^{-(E(Q_{\text{TH}}))^{-1} y} \tag{8}$$

$$E(Q_{\text{TH}}) \quad \frac{I v_{\text{TH}}}{2 s_{\text{TH}}}\left[\frac{s_{\text{TH}} C}{s_{\text{TH}} g_{\text{TH}} - v_{\text{TH}} C}\right] \tag{9}$$

where

- v_{TH} = average through arrival rate during the entire cycle,
- I = appropriate variance-to-mean ratio for arrivals and departures, and
- r_{TH} = red time for the through traffic.

Obviously, with the technique of diffusion approximation, $F_Q(y)$ is a continuous function of y. However, of interest is the discrete distribution of N_q, the residual queue on the adjacent through lane of a left-turn bay. Because N_q can be obtained by dividing Q_{TH} by the number of through lanes, one can convert $F_Q(y)$ into a discrete type as the following equation shows:

$$\Pr(N_q = n) = F_Q((n+1)l) - F_Q(nl) \tag{10}$$

where l is the number of through lanes. Because the focus is on the length of queues with relative high probability in practice, one can use the truncated distribution of Equation 10.

One may still be concerned with the influence of blocked left-turning vehicles to the residual queue and consequently through traffic delay. In fact, it cannot cause any large error because during heavy traffic the through delay caused by blocked left-turning vehicles should be small compared with total delay. Let us consider the number of blocked vehicles as the fluctuations in the number of through traffic and investigate its influence on delay. Note that during heavy traffic, at most cycles, the processes of departures and arrivals are uncorrelated. By mimicking the methods in Newell's work (*7*), one can show the fraction of the average invoked errors d^T_ϵ to the average total through delay d^T_{TH} as follows:

$$\frac{d^T_\epsilon}{d^T_{\text{TH}}} = \frac{1}{r_{\text{TH}}\left(s_{\text{TH}} - v_{\text{TH}}\right)} = O\left(\frac{1}{s_{\text{TH}} g_{\text{TH}}}\right) \tag{11}$$

where r_{TH} is red time for through traffic and $O(\cdot)$ is big O notation meaning the same order. In heavy traffic, this term is small, and therefore, the error terms contributed by blocked vehicles can be disregarded. From this analysis, the total uniform delay for through traffic per cycle can be estimated as

$$d^T_{\text{TH}} = \frac{s_{\text{TH}} v_{\text{TH}} r^2_{\text{TH}}}{2\left(s_{\text{TH}} - v_{\text{TH}}\right)} + CE(Q_{TH}) \tag{12}$$

Equation 13 is used to obtain the average uniform delay per vehicle per cycle for an entire intersection.

$$d = \frac{d^T_{\text{TH}} + d^T_{\text{LT}}}{\left(v_{\text{TH}} + v_{\text{LT}}\right)C} \tag{13}$$

Equation 13 can be used to replace the uniform delay in the HCM methods to estimate the control delay.

Delay Model for Lagging Protected Left-Turns in Heavy Traffic

For the lagging protected left-turn signal during high demand, the left-turning vehicles are most likely to spill out of the bay rather than be blocked by through vehicles. This phenomenon would occur for some cycles when left-turn flow is close to capacity. If the left-turning vehicles spill out of the bay, there would be some vehicles left over until the next cycle and the left-turn spillover queue would block adjacent through traffic. Hence, the through capacity would be reduced and total delay increased during the through green phase. Because left-turn queues are not significantly influenced by through flow during spillback, similar to the analysis of through traffic, one can apply Equations 8 and 9 to left-turn residual queues, with some adjustment to the saturation rate and arrival rate. Accordingly, the total delay can be calculated by an adjustment to Equation 12. However, Equation 12 is accurate only when left-turn flow is close to capacity. If the left-turn traffic is not close to capacity, then some corrections would be needed to Equation 10. Refer to Equation 35 in work by Newell (*7*).

First, one calculates the probability of left-turn spillback from the bay, which is a combination of two events: at least $N + 3 - Q_{\mathrm{LT}}$ left-turning vehicles arrive at the intersection and the adjacent through vehicles are not able to block the bay. Here, it is assumed that the transitional area between the left-turn bay and the through lane can contain two vehicles. Thus, if the left-turn residual queue is denoted by Q_{LT}, the spillback probability is estimated as

$$P_{\mathrm{spill}} = \sum_{m,n=0}^{N+1} \Pr\left(\left\{X_{\mathrm{LT}} \geq N + 3 - m\right\} \cap \left\{X_{\mathrm{TH}} \leq N + 1 - n\right\}\right) \Pr\left(Q_{\mathrm{LT}} = m\right) \Pr\left(N_q = n\right) \tag{14}$$

In estimating the probability in Equation 14, one encounters the same problem encountered in Equation 1. The method suggested in the discussion for Equation 1 can be applied to this equation. For an isolated intersection, Poisson distribution is a good approximation.

When there is left-turn spillback, the adjacent through lane is blocked and the through capacity is reduced accordingly. The through vehicles should wait behind the spilled left turns if they cannot seek a chance to move to an unblocked through lane. In this study, the authors consider only the case that the through volume is less than the reduced capacity and leave the other case to future work. In addition, the authors assume the blocked through vehicles on the adjacent lane wait until no spillback occurs. Therefore, the delay for through vehicles in this situation can be estimated from two parts, one accounting for the queue departure delay caused by the reduced capacity and the other accounting for the waiting time for the blocked through vehicles during the left-turn red time. Because the number of lanes possibly used for through traffic reduces to $l - 1$ during spillback, the total delay for through traffic can be calculated as follows:

$$d_{\mathrm{TH}}^{T} = P_{\mathrm{spill}} d_{\mathrm{spill}}^{T} + \left(1 - P_{\mathrm{spill}}\right) d_{\mathrm{nonspill}}^{T} \tag{15}$$

$$d_{\mathrm{spill}}^{T} = \frac{s_{\mathrm{TH}} v_{\mathrm{TH}} r_{\mathrm{TH}}^{2} \left(l-1\right)/l}{2\left(s_{\mathrm{TH}}\left(l-1\right)/l - v_{\mathrm{TH}}\right)} + \frac{1}{2}\frac{v_{\mathrm{TH}}}{l} r_{\mathrm{LT}}^{2} \tag{16}$$

where the calculation of $d_{\mathrm{nonspill}}^{T}$ is similar to Equation 5. The total uniform delay d_{spill}^{T} can also be converted to the average delay per vehicle to replace the uniform model in the HCM methods.

MODEL VALIDATION

In the following subsections, the authors first calibrate the simulation using saturation flow rate and observed queue lengths. The parameters in simulation are set according to the basic settings (*15*), in which the simulation was calibrated according to field data. The authors then examine the proposed left-turn delay model under a leading left-turn operation with comparison with the HCM results and the simulation during high through traffic demand. Then the proposed through delay model for a lagging left-turn operation during high left-turn traffic demand is evaluated.

Simulation Setup

A two-lane isolated signalized intersection is set up in VISSIM (*17*), and the simulation data are generated with different scenarios for the evaluation of the proposed model. The intersection operates with protected left turns and has ideal conditions: two 12-ft through lanes with one left-turn bay, all passenger cars, no parking, and no pedestrians. The length of the left-turn bay was selected as a variable in the capacity calculation. Because in VISSIM the length of passenger cars varies from 13.48 to 15.62 ft, the length of the bay was adjusted by observing the number of vehicles during a simulation. The basic calibrated data for the delay calculation follow:

- Protected left-turn green: 13 s,
- Through vehicle green: 50 s,
- Through vehicle red and change time: 56 s,
- Total cycle length: 106 s,
- Through vehicle saturation flow rate: 1,800 vehicles per hour (vph), and
- Protected left-turn saturation flow rate: 1,700 vph.

Different through volumes and left-turn volumes are set for the leading and lagging left-turn operations, aiming to enhance the left-turn phenomena for different strategies. In addition, because there is no choice to directly control the saturation flow rate in VISSIM, the authors changed some environment setups to calibrate these values. The saturation flow rate was obtained by averaging the outcome of lane throughput from 15 multiple runs, each of which lasted 100 s, for discharging queued vehicles under a fully congested situation.

To manage the stochastic nature of VISSIM, 15 simulation runs for each of seven length scenarios of left-turn bays, 105 in total, were conducted for the leading and lagging left-turn operations by changing the random number seeds in VISSIM. Each run lasted 1 h with increments of 15 min, and the highest 15-min delay was chosen to compute the average control delay for each scenario. The reason for this choice is that the control delay model in the HCM is based on the highest flow level of different 15-min periods.

Validation of Proposed Left-Turn Delay Model for Leading Left-Turn Operation

In VISSIM, the through volume is set to 1,650 vph and the left-turn volume is set to 100 vph under a leading left-turn operation. In this case, the through demand is high. Hence, the residual queue problem becomes a concern, resulting in the high possibility of blockage of the left-turn bay. Zhang and Tong (*15*) observed the blockage in the field, especially when the left-turn bay is short. For a longer left-turn bay, the left-turn delay is expected to be smaller because the chance of blockage should be lower. In simulation, the delay for left-turn traffic varies from 69 s per vehicle for a left-turn bay length of five vehicles to 55 s per vehicle for a bay length of 11 vehicles, as shown in Figure 2. This result is generally consistent with the expectation. However, the left-turn delay for the bay length of eight vehicles

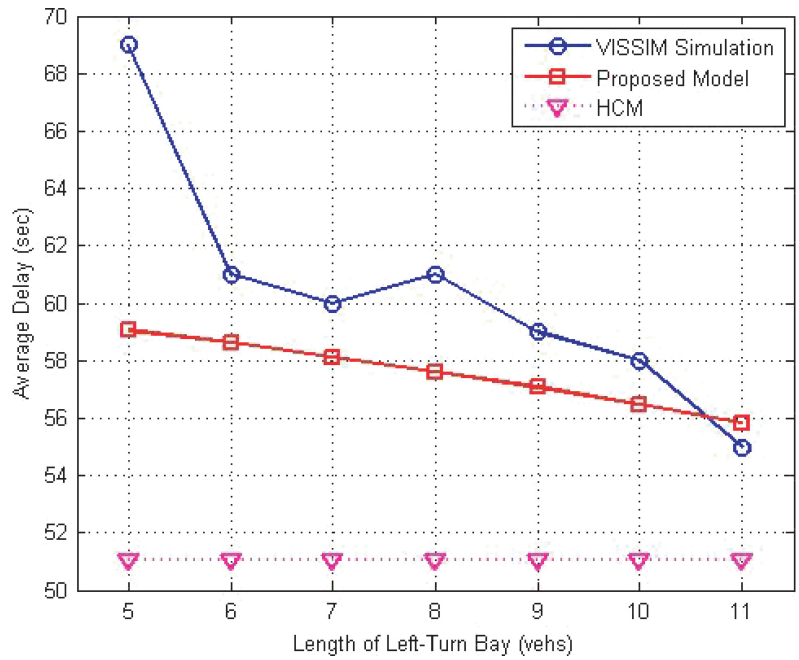

FIGURE 2 Comparison of left-turn delays under leading left-turn operation.

is a little larger than the ones for the bay lengths of seven and nine vehicles. As shown in Table 1, the reason can be attributed to the standard deviation of delay in the sample simulation data. Within the 15 simulation runs, the standard deviation of left-turn delay for the bay length of eight vehicles is larger than the others. Nevertheless, it does not mean that the former delay is actually longer. Such results are due to the stochastic nature of arrivals. For this case, a reasonable explanation is that there is no significant difference among the delays for the bay lengths of seven to nine. Moreover, observations from the field and the simulations show that blockage occurs when the bay is short, and left-turn spillback can also occur. The blocked adjacent through vehicles increase the chance of the longer residual queues and blockage of the left-turn bay would be most likely to occur. Such complex phenomena cause a larger left-turn delay for shorter bays. As shown in Figure 2, the simulation result of a larger delay for the bay length of five vehicles than for the bay length of six vehicles is due to this reason.

The proposed models for left-turn delay, Equations 3 to 6, are used to calculate the control delay by replacing the uniform delay term in the HCM delay model (Chapter 16 in the HCM). Regarding the increment term in HCM model, the recommended values for isolated intersections are used to set the parameters (*2*). The progression adjustment factor was set to one because of the isolated intersection. No initial queue delay term in the HCM methods was added. As Figure 2 shows, the comparison of all results demonstrates that the proposed left-turn delay model estimates well the increase of left-turn delay caused by the blockage of through traffic. It is obvious that the HCM methods significantly underestimate the left-turn control delay in this case. However, because the proposed model does not consider the complex phenomena discussed previously for short left-turn bays, there is a gap between the simulation result and the proposed model for the bay length of five vehicles. Such inaccuracy will be considered in future work.

Another case with a left-turn volume of 160 vph and a through volume of 1,500 vph was used to validate the proposed left-turn delay model. In this case, the volume-to-capacity ratio for left-turn traffic is as high as 0.76. As shown in Figure 3, the results demonstrate the merits of the proposed model.

TABLE 1 Standard Deviation of Left-Turn Delay in Simulation

Bay Length	Standard Deviation of Delay
5	3.83
6	5.04
7	3.46
8	7.45
9	6.09
10	4.58
11	3.98

Validation of Proposed Through Delay Model for Lagging Left-Turn Operation

In the simulation, the through volume is set to 1,300 vph and the left-turn volume is set to 205 vph under a leading left-turn operation. The left-turn volume is quite close to the saturation flow rate. Therefore, overflows of left-turn traffic are expected for some cycles, resulting in left-turn spillback from bays of insufficient length. Such a phenomenon was observed from the field as well (*15*). The delay for through traffic varies from 48 s per vehicle for a left-turn bay length of five vehicles to 34 s per vehicle for a bay length of 11 vehicles, as shown in Figure 4. Generally, it is consistent with intuition as well to say that the delay decreases with respect to the increase of bay length. However, the through delay for a bay length of seven

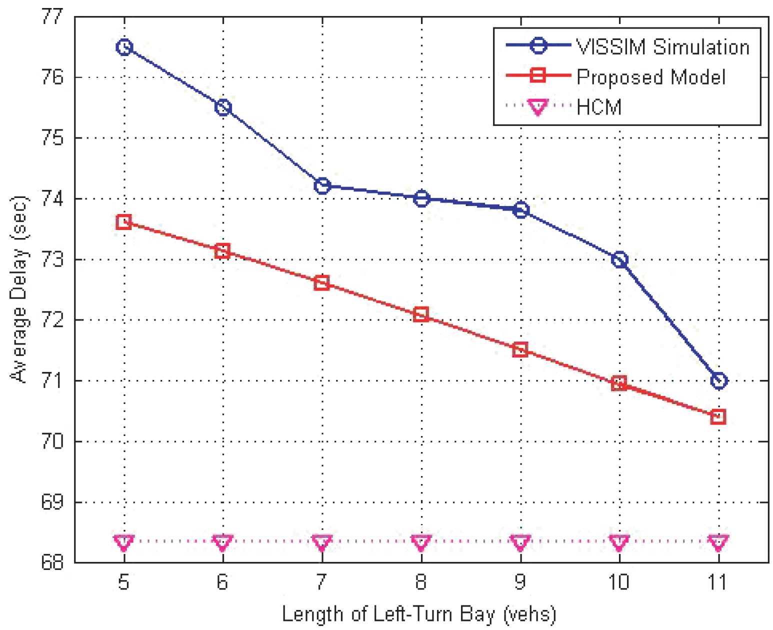

FIGURE 3 Comparison of left-turn delays with left-turn volume of 160 vph and through volume of 1,500 vph.

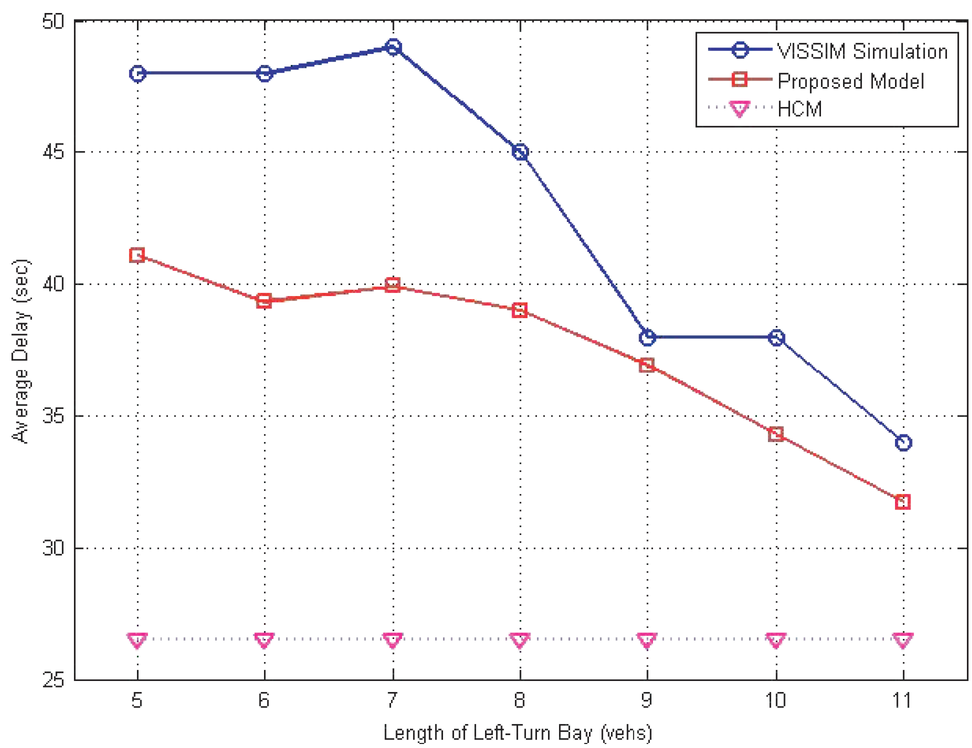

FIGURE 4 Comparison of through delays under lagging left-turn operation.

vehicles is a little larger than the one for a bay length of six vehicles. This phenomenon is similar to that of the leading left-turn operation. It does not mean that the former delay is actually longer but is attributable to randomly obtained samples. It is likely to occur when one deals with field data as well. For this case, it indicates these two delays are almost the same.

The proposed models for through delay, Equations 15 and 16, are used to calculate the control delay by replacing the uniform delay term in the HCM delay model. Figure 4 shows the results from the proposed models and the HCM methods. The comparison of all results demonstrates that the proposed through delay model reflects well the increase of delay caused by the left-turn spillback. Obviously, the HCM methods significantly underestimate the through control delay in the case of left-turn spillback. However, there is still a gap between VISSIM simulation and the proposed model when the left-turn bay is not long. The gap is partially due to the longer queue of left-turn spillback for a shorter bay. Consequently, the vehicles in the blocked through lane are much harder to get out of the bottleneck even if the drivers get the chance to change to the right lane. The proposed model does not count this issue. It is also noticed that the proposed through delay with the bay length of six vehicles is slightly lower than that of the bay length of seven. The difference is because of the methods of estimating spillback probability in Equation 14. When the left-turn bay is relatively short, the independent treatment of through and left-turn arrivals is not accurate. These issues will be studied in the future.

CONCLUSION AND FUTURE WORK

This paper studies the uniform delay with leading and lagging protected left-turn operations in heavy traffic. Delay is inherently related to blockage of the left-turn bay, left-turn spillback, and residual queues at a signalized intersection, but these factors are not considered in existing delay models. The main contribution of this study is a proposed left-turn delay model for the leading signal strategy and a proposed through delay model for the lagging strategy to capture the complex phenomena between the left-turn traffic and adjacent through traffic. The proposed methods are based on the complex interactions between left-turning and through vehicles.

Although the methods appear complicated, one cannot evaluate vehicle delay at a signalized intersection without some constructive methods. The comparison of the results of the proposed models, the HCM methods, and the VISSIM simulations demonstrates the merits of the proposed models under heavy traffic. There are several directions to improve the delay models with respect to left-turn traffic. Future work should further investigate the probability of blockage to the left-turn bay and spillback of left-turn traffic. Such investigation is critical to further improve the accuracy of the proposed models. The robustness of the model will be studied in terms of the time-varying arrival rates and the number of "sneakers." The proposed models should also be validated with field data and the issues, such as the delay for general arrival patterns and the control delay for both left-turn and through traffic in a system of signalized intersections, should also be further investigated.

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The Highway Capacity and Quality of Service Committee peer-reviewed this paper.